Journal of Applied Mechanics and Technical Physics, Vol. 42, No. 2, pp. 276–278, 2001

MOTION OF A CIRCULAR CYLINDER IN A VIBRATING LIQUID

I. E. Kareva and V. L. Sennitskii

UDC 532.582

The motion of a circular cylinder under gravity in an ideal liquid bounded from the outside by a vibrating wall is determined using numerical methods.

The motion of an absolutely rigid circular cylinder under gravity in an ideal incompressible liquid bounded from the outside by a planar, absolutely rigid, vibrating wall was considered previously [1]. In that work, it was established that in a vibrating liquid, a rigid body can behave paradoxically: a cylinder whose density is different from the liquid density, can, in particular, be at rest on the average. The theoretical study [1] of the paradoxical behavior of a rigid body in a vibrating liquid under gravity was continued in [2, 3] (see also [4]). In [1, 3], analytical solutions of the problems of motion of rigid bodies (circular cylinder and sphere) in a liquid were obtained under the assumption that the distance between the body and the wall is large compared to the radius of the body. The present paper reports results of investigation of the problem of [1] using numerical methods for the case where the distance between the cylinder axis and the wall surface is insignificant compared to the radius of the cylinder.

1. We consider the problem in the formulation given in [1]. We assume that x and y are inertial rectangular coordinates in the flow plane (Fig. 1), a is the radius of the cylinder, $O(L, 0)$ is the point of intersection of the flow plane with the cylinder axis, h ($h > a$) is the distance between the point O and the line of intersection of the flow plane with the wall surface, $H = L - h$, $\rho_{\rm cyl}$ is the density of the cylinder, $\rho_{\rm liq}$ is the liquid density, and $\boldsymbol{g} = (-g, 0)$ is the free-fall acceleration.

The equation of motion for the cylinder and the initial conditions in the coordinate system $\hat{x} = x - H$, $\hat{y} = y$ attached to the wall have the form

$$\frac{d^2 h}{dt^2} = \frac{F}{\pi a^2 \rho_{\rm cyl}} - g - \frac{d^2 H}{dt^2}, \tag{1}$$

$$h = h_0, \qquad \frac{dh}{dt} = 0 \quad \text{for} \quad t = 0, \tag{2}$$

where t is time, h_0 is a constant, and $F = \pi a^2 \rho_{\rm liq}(g + d^2H/dt^2 + f_1\, d^2h/dt^2 + f_2 a^{-1}(dh/dt)^2)$ is the force exerted by the liquid on unit length of the cylinder along the $\hat{x}$ axis, which is found in [1]. Here

$$f_1 = -4\sinh^2\eta_0 \sum_{m=1}^{\infty} a_m, \quad f_2 = 2\sinh\eta_0 \sum_{m=1}^{\infty} b_m + 4\sinh\eta_0(\cosh^2\eta_0 + 1)\sum_{m=1}^{\infty} c_m - \frac{\cosh\eta_0}{\sinh^2\eta_0},$$

where

$$a_m = m\,\mathrm{e}^{-2m\eta_0}\coth m\eta_0, \quad b_m = m\,\mathrm{e}^{-2m\eta_0}\coth m\eta_0[m\coth m\eta_0 + (m+1)\,\mathrm{e}^{-\eta_0}\cosh\eta_0\coth(m+1)\eta_0],$$

$$c_m = m\,\mathrm{e}^{-2m\eta_0}(m - \coth\eta_0)\coth m\eta_0, \quad \eta_0 = \ln\frac{h + \sqrt{h^2 - a^2}}{a}.$$

Lavrent'ev Institute of Hydrodynamics, Siberian Division, Russian Academy of Sciences, Novosibirsk 630090. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 42, No. 2, pp. 103–105, March–April, 2001. Original article submitted May 29, 2000; revision submitted October 5, 2000.

 0021-8944/01/4202-0276 $25.00

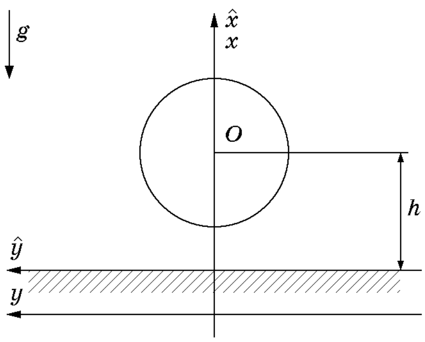

Fig. 1

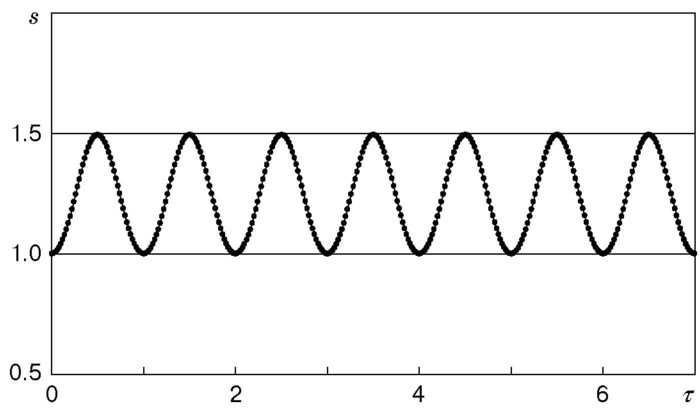

Fig. 2

TABLE 1

β	ε_-	ε_+	$\langle s\rangle$
1.09	0.92028694	0.92028695	1.2389
1.16	0.93397724	0.93397725	1.2398
1.25	0.94949407	0.94949408	1.2405
1.34	0.96285645	0.96285646	1.2408
1.45	0.97650449	0.97650450	1.2405
1.57	0.98825821	0.98825822	1.2397

2. Let

$$H = A_0\Big(1 - \cos\frac{2\pi t}{T}\Big), \tag{3}$$

where A_0 ($A_0 > 0$) and T ($T > 0$) are constants.

According to (1)–(3), we have

$$A\,\frac{d^2s}{d\tau^2} + B\Big(\frac{ds}{d\tau}\Big)^2 + C = 0, \tag{4}$$

$$s = 1, \qquad \frac{ds}{d\tau} = 0 \quad \text{for} \quad \tau = 0, \tag{5}$$

where $\tau = t/T$, $s = h/h_0$, $A = f_1 - \rho$, $B = f_2/\varepsilon$, and $C = \varepsilon(1-\rho)(\beta + 4\pi^2\alpha\cos 2\pi\tau)$ ($\alpha = A_0/a$, $\beta = gT^2/a$, $\varepsilon = a/h_0$, and $\rho = \rho_{\rm cyl}/\rho_{\rm liq}$).

Problem (4), (5) reduces to the problem

$$\frac{d\boldsymbol{Q}}{d\tau} = \boldsymbol{F}, \tag{6}$$

$$\boldsymbol{Q} = \boldsymbol{Q}_0 \quad \text{for} \quad \tau = 0, \tag{7}$$

where

$$\boldsymbol{Q} = \begin{pmatrix} s \\ q \end{pmatrix}, \quad \boldsymbol{Q}_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \boldsymbol{F} = \begin{pmatrix} q \\ f \end{pmatrix} \quad \Big[q = \frac{ds}{d\tau} \quad \text{and} \quad f = -\frac{1}{A}\,(Bq^2 + C)\Big].$$

3. Problem (6), (7) was solved numerically using an explicit three-stage Runge–Kutta method with fourth-order approximation in a step [5]. Values of f were calculated from the first K terms of the series in the expressions for f_1 and f_2, and the estimates of the residues of these series obtained in [1] were used. In the calculation of the motion of the cylinder, the parameters had the following values: $\alpha = 1$, $\beta = 1.09$, 1.16, 1.25, 1.34, 1.45, and 1.57, $\rho = 0.5$, $K = 60$, and $d = 0.001$.

For various values of β, we obtained intervals that contain the values of ε for which the cylinder is at rest on the average, vibrates about the position $\langle s \rangle = \int_0^1 s\, d\tau$, and is in the state of paradoxical equilibrium (see Table 1).

Figure 2 gives a curve of s versus τ for $\beta = 1.57$ and $\varepsilon = 0.98825821$.

4. The calculations performed lead, in particular, to the conclusion that the paradoxical behavior of a circular cylinder in a vibrating liquid, established in [1] for the case where the ratio of the cylinder radius to the distance between the cylinder axis and the wall surface is small compared to unity, also occurs for large values of this ratio compared to unity.

REFERENCES

1. V. L. Sennitskii, "Motion of a circular cylinder in a vibrating liquid," *Prikl. Mekh. Tekh. Fiz.*, No. 5, 19–23 (1985).
2. B. A. Lugovtsov and V. L. Sennitskii, "Motion of a body in a vibrating liquid," *Dokl. Akad. Nauk SSSR*, **289**, No. 2, 314–317 (1986).
3. V. L. Sennitskii, "Motion of a sphere in a liquid in the presence of a vibrating wall," *Prikl. Mekh. Tekh. Fiz.*, **40**, No. 4, 125–132 (1999).
4. V. L. Sennitskii, "Motion of inclusions in a vibrating liquid," *Sib. Fiz. Zh.*, No. 4, 18–26 (1995).
5. I. S. Berezin and N. P. Zhidkov, *Methods of Calculations* [in Russian], Vol. 2, Fizmatgiz, Moscow (1962).